ABSTRACT:

These lectures will describe a new class of variational problems, originally motivated by problems of optimal confinement of wild fires.

The area burned by the fire (or contaminated by a spreading chemical agent) at time $t > 0$ is modelled as the reachable set for a differential inclusion $\dot{x} \in F(x)$, starting from an initial set R_0. We assume that the spreading of the contamination can be controlled by constructing walls (or barriers). These are rectifiable curves $\gamma(t)$ whose length is allowed to grow linearly in time.

After a precise description of the mathematical model, the first lecture will examine under which conditions one has existence (or non-existence) of a strategy that blocks the fire within a bounded domain, completely surrounded by walls.

The following lectures will be devoted to the optimization problem. Here we assign functions $\alpha(x)$ describing the unit value of the land at the location x, and $\beta(x)$ accounting for the cost of building a unit length of wall near x. We then seek an optimal strategy, minimizing the total value of the burned region, plus the cost of building the barrier. A general theorem on the existence of optimal strategies will be presented, together with some necessary conditions for optimality. Several related questions and open problems will be discussed.