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Special exact solutions of a nonlinear diffusion equation*

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The report is devoted to the construction of compactly supported solutions of the nonlinear diffusion equation with a power-law diffusivity. We present the results of a qualitative analysis of these solutions.

Keywords: compactly supported solutions, nonlinear diffusion equation, qualitative analysis

We consider the quasilinear parabolic partial differential equation

$$\partial_t u = \nabla_x \cdot [k(u)\nabla_x u], \tag{1}$$

usually called the *nonlinear diffusion equation.* Here $u = u(t,x)\colon \Omega \to R_{\geq 0}$, $\Omega \subset R_{\geq 0} \times R^n$, $n \in N$ is the density of the diffusing material at location $x = (x_1, \dots, x_n)$ and time t; $k(u) = k_0 u^\sigma$ is the diffusion coefficient and k_0, σ are real positive parameters (slow diffusion). Generally speaking, eq. (1) also describes many processes that occur in problems of heat transfer, theory of combustion and explosion, fluid and gas filtration, chemical kinetics, etc.

Eq. (1) belongs to the class of degenerate, since coefficient $k(u)|_{u=0} = 0$. This degeneracy results in the interesting phenomenon [1,2]: existence of compactly supported (for all time $t \geq 0$) solutions $u(t,\,\cdot\,)$ of (1). These solutions consist of two hypersurfaces: $u = \varphi(t,x) > 0$, $\varphi \in C^{1,2}_{tx}(\Omega) \cap C(\Omega)$ (perturbed solution) and $u \equiv 0$ (unperturbed background), which are continuously joined along some hypersurface $s(t,x) = 0$ (free boundary) in $R_{\geq 0} \times R^n$. Fundamental examples of this type solutions of (1) was obtained Ya.B. Zel'dovich, A.S. Kompaneets and G.I. Barenblatt (ZKB solutions) [2].

This report will discuss the construction new exact solutions with compact support of the nonlinear diffusion equation (1) by using the Clarkson–Kruskal direct method [3] and qualitative properties of these solutions, obtained with methods of dynamical systems theory [4] and power geometry [5].

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